

Exam. Code : 103203
Subject Code : 1107

B.A./B.Sc. 3rd Semester
MATHEMATICS

Paper—I
(Analysis)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— There are *eight* questions. Candidates are required to attempt any *five* questions. All questions carry equal marks.

SECTION—A

- Prove that a monotonically decreasing sequence is convergent if and only if it is bounded below. Moreover it converges to g.l.b. of range.
 - Prove that sequence defined by $x_1 = \sqrt{7}, x_{n+1} = \sqrt{7+x_n}$ is convergent and converges to positive root of $y^2 - y - 7 = 0$.
- State and prove Cauchy's second theorem on limits.
 - If $x_1 = \frac{1}{2}\left(x + \frac{a}{x}\right), x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for all $n \in \mathbb{N}$ where $x, a > 0$ then show that $\lim_{n \rightarrow \infty} x_n = \sqrt{a}$.

SECTION—B

3. (a) Test the convergence and divergence of the series

$$1 + \frac{2}{1} \frac{1}{2} + \frac{2}{1} \frac{4}{3} \frac{1}{3} + \frac{2}{1} \frac{4}{3} \frac{6}{5} \frac{1}{4} + \dots$$

- (b) If $a_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}} = l$ then show that series

$\sum_1^{\infty} a_n$ is convergent if $l > 1$ and divergent if $l < 1$.

4. (a) Prove that sum of absolutely convergent series is independent of the order of terms.

- (b) Discuss the convergence of the series

$$1 + \frac{2x}{3^2} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \dots \infty, x > 0$$

SECTION—C

5. (a) Prove that every monotonic function on $[a, b]$ is Riemann integrable on $[a, b]$.

- (b) If $f(x) = \begin{cases} \cos x & \text{when } x \text{ is rational} \\ \sin x & \text{otherwise} \end{cases} \quad 0 \leq x \leq \pi/4,$

Is f Riemann integrable on $[0, \pi/4]$?

6. (a) Prove that if $f(x) = 2x+3$, $0 \leq x \leq 2$ then f is Riemann integrable
- (b) Prove that if f is bounded function on $[a, b]$ then $f \in R[a, b]$ iff for given $\epsilon > 0$ there exist a partition P of $[a, b]$ that $U(P, f) - L(P, f) < \epsilon$.

SECTION—D

7. (a) Evaluate :

$$\int_0^{\infty} \frac{x^2}{1+x^4} dx.$$

- (b) Test the convergence of $\int_0^{\pi} \frac{\sqrt{x}}{\sin x} dx$.

8. (a) Prove that $\beta(l, m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

- (b) Show that :

$$\int_0^{\pi} x^m (\csc x)^n dx \text{ exist if and only if } n < m + 1$$